A NEW ODONTOGRAPH.

BY GEORGE B. GRANT, Boston, Mass.

My object is to explain the construction and operation of an odontograph, or rather, an odontographic method, which is not only arranged with special reference to convenience and ease of operation, but which is also as accurate as can be desired for all ordinary purposes of construction or drafting.

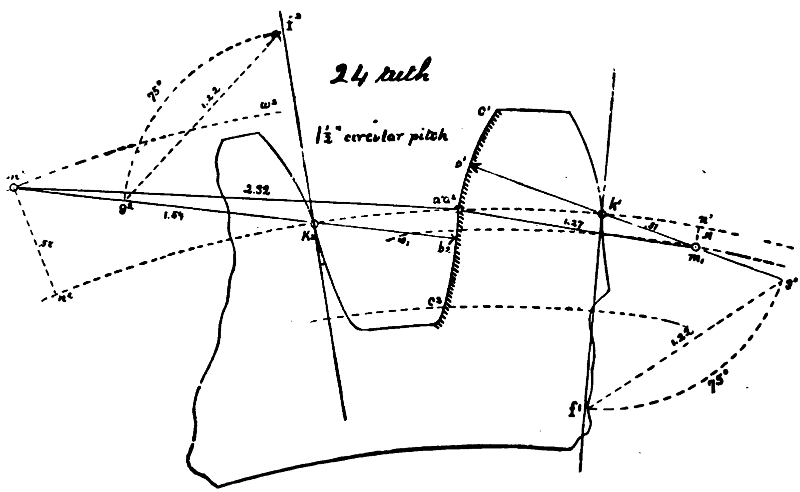

FIG. 1. Willis Odontograph.

In my business, which is specially devoted to the construction of gearing, I formerly had frequent occasion to make use of the well-known Willis odontograph, which has practically been the only one in use for thirty years past.

When using this instrument, the method, as illustrated by *Fig. 1*, is as follows: (1.) The pitch circle is divided off for the pitch points. (2.) A radius is drawn to the pitch circle. (3.) The instrument is laid on the radius. (4.) The positions on the instrument, of the centres, are found by the use of a table. (5.) Circles are drawn through the centres. (6.) The tooth-curves are all drawn in from centres on the centre-circles.

It occurred to me, after wading through this process many times, that much of the work could be got rid of by calculating the

results of the process and arranging them in a table. The real object in view, in the process outlined above, is to get the positions of the circles that contain the centres, and the lengths to set up on the dividers when drawing the tooth-curves. I constructed and for a time used a table giving the distances $m^1 n^1$ and $m^2 n^2$ of the circles of centres, and the radii $a^1 m^1$ and $a^2 m^2$ for the tooth curves. The result was that much of the labor was avoided and the accuracy of the result greatly improved. It is a work of great skill to so handle the Willis instrument that the same example will give the same result when the operation is repeated, while the use of the table will give uniform results without trouble. The

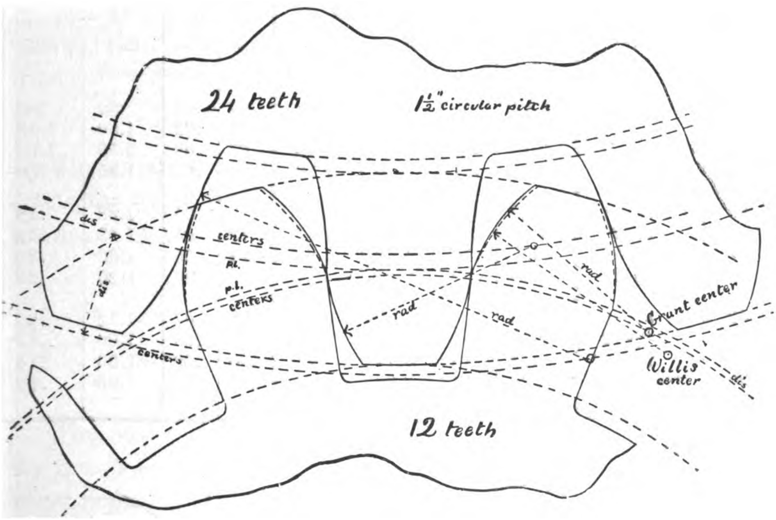

FIG. 2. Epicycloidal Teeth.

two methods are identical as to the position of the resulting tooth-curve.

It afterwards occurred to me that the process would be in no way lengthened, or its difficulty increased, if the centres, found by the use of the table, were the centres of the circles which most nearly coincide with the true curves, instead of the Willis centres of the osculatory circles at the centres of the curves, and this is the method here given and illustrated by *Fig. 2* and the Table I.

The method adopted for the construction of the table was a mathematical process, tested by graphical processes. I first computed the co-ordinates of the points of the true curve at the adden-

dum point and at the centre of the curve, then the co-ordinates of the centres of the circles that pass through these two points and through the pitch points, and then the actual lengths of the radii of the circles as well as the distances of the centres of circles from the

GRANT'S ODONTOGRAPH TABLE I.—EPICYCLOIDAL TEETH.

INTERCHANGEABLE SERIES.

From a Pinion of Twelve Teeth to a Rack.

NUMBER OF TEETH IN THE GEAR.		FOR ONE DIAMETRAL PITCH. For any other pitch, divide by that pitch.				FOR ONE INCH CIRCULAR PITCH. For any other pitch, multiply by that pitch.			
		FACES.		FLANKS.		FACES.		FLANKS.	
Exact.	Intervals.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.
12	12	2.01	.06	∞	∞	.64	.02	∞	∞
13½	13–14	2.04	.07	15.10	9.43	.65	.02	4.80	3.00
15½	15–16	2.10	.09	7.86	3.46	.67	.03	2.50	1.10
17½	17–18	2.14	.11	6.13	2.20	.68	.04	1.95	.70
20	19–21	2.20	.13	5.12	1.57	.70	.04	1.63	.50
23	22–24	2.26	.15	4.50	1.13	.72	.05	1.43	.36
27	25–29	2.33	.16	4.10	.96	.74	.05	1.30	.29
33	30–36	2.40	.19	3.80	.72	.76	.06	1.20	.23
42	37–48	2.48	.22	3.52	.63	.79	.07	1.12	.20
58	49–72	2.60	.25	3.33	.54	.83	.08	1.06	.17
97	73–144	2.83	.28	3.14	.44	.90	.09	1.00	.14
290	145–rack.	2.92	.31	3.00	.38	.93	.10	.95	.12

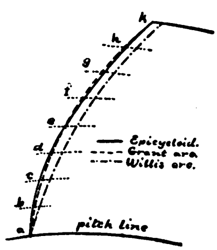

FIG. 3.

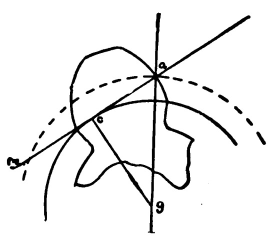

FIG. 4.

pitch lines. It is obvious that these circles are as close to the true curve as circular arcs can be made.

The improvement made in accuracy is illustrated in *Fig. 3.* The new arc crosses the curve twice, coinciding with it at three points, while the Willis arc leaves it at the pitch line and thence

runs entirely inside of it. The following table gives the errors of both arcs at nine points of a twelve-tooth pinion of a diametral pitch of unity, computed with great care to four places of decimals, or ten-thousandths of an inch. It is seen that the average error of the old curve is about six times that of the new.

	New Curve.	*Willis.*
At *a*	·0000	+·0000 inches.
" *b*	+ ·0088	+·0175 "
" *c*	+ ·0091	+·0244 "
" *d*	+ ·0056	+·0283 "
" *e*	·0000	+·0288 "
" *f*	— ·0036	+·0297 "
" *g*	— ·0061	+·0308 "
" *h*	— ·0046	+·0342 "
" *k*	·0000	+·0397 "
Average	± ·0042	+ ·0260

The greatest error of the new arc is at about the quarter point *c*, and for a tooth of a diametral pitch of unity, twice the size of the tooth of *Fig. 2*, this maximum error is as follows:

For	12 teeth	$c =$ ·009	inches.
"	20 "	$c =$ ·008	"
"	40 "	$c =$ ·006	"
"	100 "	$c =$ ·004	"
"	300 "	$c =$ ·002	"

The new curve is the most accurate, approximating most nearly to the true curve, at the point of the tooth, where accuracy is most needed, and where the Willis arc is most at fault.

Fig. 2 shows the two processes applied to a twelve-tooth pinion, the full arcs being by the new process and the dotted arcs by the Willis process. As the number of teeth increases, the accuracy of both arcs improves, until, for a very large gear either arc practically agrees with the true arc.

It may reasonably be claimed that the new arc is as near to the true curve as it is necessary to get, for hand and eye processes cannot appreciate a greater accuracy. It is doubtful if anything but the most delicately-constructed shaping engine could form a templet with a smaller error, for it is found in practice that templets constructed with the greatest skill by ordinary means, must be brought into shape by hand and eye processes, before they are

ready for use for the purpose of making gear cutters. The writer does not know of a purely mechanical process used in the construction of gear-teeth that will give closer results, except the Bilgram originating machine.

The Beale epicycloidal engine of the Brown & Sharpe Manufacturing Company will shape theoretically correct templets and templet gears with the greatest precision, the errors being all within a thousandth of an inch in magnitude, but, unlike the Bilgram machine, it has not as yet been applied directly to the shaping of the teeth of working wheels. But machines of this class can hardly

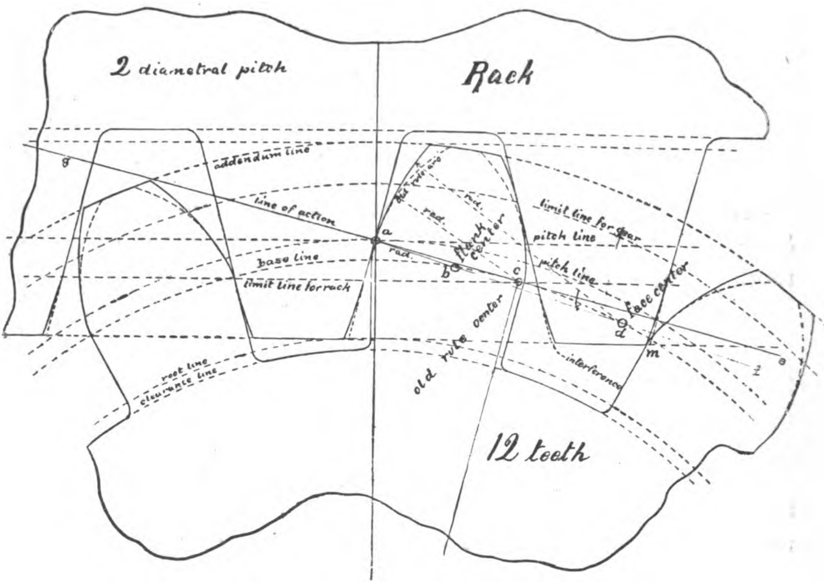

FIG. 5. Involute Teeth.

be compared with odontographs, as their expense puts them beyond reach for ordinary drafting or constructive purposes.

The principal application is to the interchangeable system of gear-teeth, but I have also applied it to the case, sometimes met with in practice, of teeth with radial flanks, not interchangeable. The upper number in each square of the Table II, is the face radius, while the lower is the centre distance. As shown by the table, most of the centres are inside the pitch line, some are on the line, while others, those having the negative sign, are outside of it.

A great advantage of the new method is that it can be applied

GRANT'S ODONTOGRAPH TABLE II.—EPICYCLOIDAL TEETH.

RADIAL FLANK TABLE.

FOR ANY POSSIBLE PAIR OF GEARS, NOT INTERCHANGEABLE.

One Inch Circular Pitch.

For any other pitch, multiply by that pitch.

NUMBER OF TEETH IN GEAR BEING DRAWN.		NUMBER OF TEETH IN THE MATE.											
Exact.	Intervals.	12	13 14	15 16	17 18	19 21	22 24	25 29	30 36	37 48	49 72	73 144	145 rack
12	12	.64 .02	.64 .01	.65 .01	.66 .01	.67 0	.68 0	.69 -.01	.70 -.01	.71 -.02	.73 -.02	.74 -.03	.75 -.03
13½	13-14	.65 .02	.66 .02	.67 .01	.68 .01	.69 .01	.70 0	.72 0	.74 -.01	.75 -.01	.76 -.02	.78 -.02	.79 -.03
15½	15-16	.67 .03	.68 .02	.69 .02	.70 .01	.72 .01	.74 .01	.75 0	.78 0	.79 -.01	.82 -.02	.84 -.02	.84 -.03
17½	17-18	.68 .04	.70 .03	.71 .02	.73 .02	.75 .01	.77 .01	.78 .01	.82 0	.84 -.01	.87 -.01	.89 -.02	.90 -.03
20	19-21	.70 .04	.72 .04	.74 .03	.76 .02	.79 .02	.81 .01	.83 .01	.87 0	.90 0	.93 -.01	.96 -.02	.96 -.03
23	22-24	.72 .05	.74 .04	.76 .04	.79 .03	.82 .02	.84 .02	.87 .01	.91 .01	.94 0	.98 -.01	1.01 -.02	1.03 -.03
27	25-29	.74 .05	.76 .05	.79 .04	.82 .04	.85 .03	.87 .02	.92 .02	.96 .01	.99 0	1.03 -.01	1.07 -.02	1.10 -.03
33	30-36	.76 .06	.79 .05	.83 .05	.86 .04	.90 .03	.94 .03	.98 .02	1.02 .01	1.06 0	1.11 0	1.17 -.01	1.23 -.02
42	37-48	.79 .07	.83 .06	.86 .05	.90 .05	.96 .04	.98 .04	1.03 .03	1.08 .03	1.14 .02	1.20 0	1.25 0	1.37 -.01
58	49-72	.83 .08	.87 .07	.91 .07	.96 .06	1.02 .06	1.05 .05	1.10 .04	1.17 .04	1.24 .03	1.30 .02	1.43 0	1.58 0
97	73-144	.90 .09	.93 .08	.97 .08	1.01 .07	1.07 .07	1.11 .06	1.18 .06	1.28 .05	1.34 .04	1.47 .03	1.65 .02	2.03 0
290	145 rack	.93 .10	.96 .09	1.00 .09	1.05 .09	1.10 .08	1.16 .08	1.24 .07	1.37 .07	1.50 .06	1.70 .04	2.12 .03	2.90 .02

GRANT'S ODONTOGRAPH TABLE III.—INVOLUTE TEETH.

INTERCHANGEABLE SERIES.

From a Pinion of Twelve Teeth to a Rack.

NUMBER OF TEETH IN THE GEAR.		FOR ONE DIAMETRAL PITCH. For any other pitch, divide by that pitch.			FOR ONE INCH CIRCULAR PITCH. For any other pitch, multiply by that pitch.		
Exact.	Intervals.	Base Distance.	Face Radius.	Flank Radius.	Base Distance.	Face Radius.	Flank Radius.
12	12	.20	2.70	.83	.06	.86	.27
13	13	.22	2 87	.93	.07	.91	.30
14	14	.23	3.00	1.02	.07	.95	.33
15	15	.25	3.15	1.12	.08	1.00	.36
16	16	.27	3.29	1.22	.08	1.05	.40
17	17	.28	3.45	1.31	.09	1.09	.43
18	18	.30	3.59	1.41	.09	1.14	.46
19	19	.32	3 71	1.53	.10	1.18	.50
20	20	.33	3.86	1.62	.10	1.22	.53
21	21	.35	4.00	1.73	.11	1.27	.57
22	22	.37	4 14	1.83	.11	1.32	.60
23	23	.39	4.27	1.94	.12	1 36	.63
25	24- 26	.42	4.56	2.15	.13	1.45	.70
28	27- 29	.45	4.82	2.37	.14	1.54	.77
31	30- 32	.50	5.23	2.69	.15	1.67	.88
34	33- 36	.57	5.77	3.13	.17	1.84	1.00
38	37- 41	.63	6 30	3.58	.19	2.01	1.16
44	42- 48	.73	7.08	4.27	.22	2 26	1.38
52	49- 58	.87	8.13	5.20	.26	2.59	1.70
64	59- 72	1.07	9.68	6.64	.32	3.09	2.18
83	73- 96	1.39	12.11	8.93	.42	3.87	2.90
115	97-144	1.92	16.18	12 80	.58	5.16	4.15
192	145-288	3.20	25.86	22.30	.96	8.26	7.30
576	289-rack	9.60	73.95	70.10	2.88	23.65	22.30

INTERFERENCE

FOR TWELVE TO RACK INTERCHANGEABLE SET.

Teeth In the gear.	12	13 14	15 16	17 18	19 21	22 24	25 29	30 36	37 48	49 72	73 144	145 ∞
Pitch.	Amount of the Interference.											
One in. cir.	.003	.007	.007	.007	.007	.007	.010	.010	.010	.013	.017	.020
One diamet'l	.01	.02	.02	.02	.02	.02	.03	.03	.03	.04	.05	.06

Interference always to commence at a point half way between pitch line and addendum line.

to the involute form of tooth as well as to the cycloidal form. The involute is superior to the cycloid, theoretically and practically, but the Willis process does not apply to it, and if drawn at all, it is generally by the ridiculously inaccurate method shown by *Fig. 4*. By this simple, but worse than worthless method, the line *a c* is drawn at an angle of 75½° with the radial line, and then the tooth-curve is drawn from a centre, *c*, at a distance *a c* of one-quarter of the radius. Nothing could be simpler, and it would be difficult to contrive anything further from the truth, for the result on a twelve-toothed pinion is shown by the dotted lines of *Fig. 5*.

The new mèthod, as applied to the involute, is given by the Table III, and illustrated by *Fig. 5*. The base circle is first drawn at the tabular distance from the pitch circle, and then the faces and flanks are drawn in with the tabular radii, from centres on the base circle. The supplementary table shows the necessary correction, for interference, on the points of the larger gears.

POPULAR ERRORS IN METEOROLOGY.

By Cleveland Abbe.

[*A Lecture delivered before the* Franklin Institute, *December 17, 1886.*]

The Lecturer was introduced by Dr. Wahl, Secretary of the Institute, and spoke as follows:

The pleasure with which I appear before you to-night for the purpose of directing your attention for a short time to some popular points in meteorology, is greatly increased by the recollection of those eminent scientists, who in this city and even in this hall, long since expounded some of the most important laws previously unknown, the knowledge of which has gradually worked a revolution in our views as to the philosophy of atmospheric processes.

You will remember that it was to Benjamin Franklin himself that we owe the general promulgation of the fact that many of our Northeast storms move slowly along our Atlantic Coast from Georgia to New England. It was here that he drew the lightning from the skies and wrested the sceptre from the hands of tyrants." To Godfrey, we owe the sextant, and to Rittenhouse the establishment of our first observatory with accurate methods of observation, both astronomical and meteorological. To Bache and the